Warm-up!

$\frac{\frac{y}{x}+1}{\frac{x}{y}}$

Which one of the following is not equivalent to -?

A)

B)

C)

D)

$= \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right) = \frac{1}{\cos\theta\sin\theta}$

$\frac{\sin\theta\ \sin\theta}{\sin\theta\ \cos\theta} + \frac{\cos\theta\ \cos\theta}{\sin\theta\ \cos\theta}$

$\frac{\sin\theta\sin\theta + \cos\theta\cos\theta}{\sin\theta\cos\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta}$

Trig Functions, Day 7

- More Identities (that we kind of already know)
- 3.10, 3.11, 3.12

Homework: Basic Identities

Quiz Class after Next (Thursday)

- Solving
- Pythag Identities
- Reciprocal Identities
- Quotient Identities
-

Pythagorean Identities

$$\frac{y^2}{x^2} + \frac{x^2}{x^2} = \frac{1}{x^2}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Example 1: Solve over the interval $0 \leq \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

$$\sin\theta + 2 = 3\csc\theta$$

$\sin\theta + 2 = 3\left(\frac{1}{\sin\theta}\right)$

$A + 2 = 3\left(\frac{1}{A}\right)$

$A(A+2) = \left(\frac{3}{A}\right) \cdot A$

$A^2 + 2A = 3$

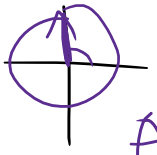
$A^2 + 2A - 3 = 0$

$(A-1)(A+3) = 0$

$A = 1$ $\quad$ $A = -3$

$\sin\theta = 1$ $\quad$ $\sin\theta = -3$ extraneous

impossible



$\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

Example 2: Solve over the interval $0 \leq \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

$$\sec\theta + 3 = -2\cos\theta$$

$$\frac{1}{\cos\theta} + 3 = -2\cos\theta$$

$$A\left(\frac{1}{A} + 3\right) = (-2A)A$$

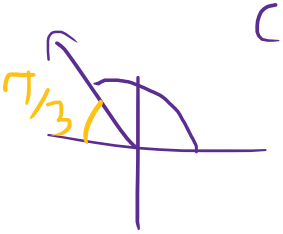
$$1 + 3A = -2A^2$$

$$2A^2 + 3A + 1 = 0$$

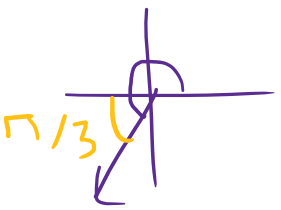
$$(2A+1)(A+1) = 0$$

$A = -\frac{1}{2}$ $\quad$ $A = -1$

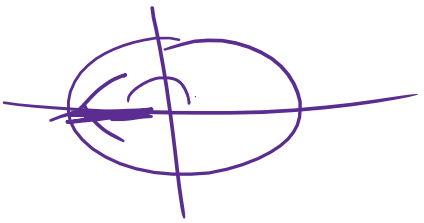
$\cos\theta = -\frac{1}{2}$ $\quad$ $\cos\theta = -1$



$\theta = \frac{2\pi}{3}$



$\theta = \frac{4\pi}{3}$



$\theta = \pi$

Example 3: Solve over the interval $0 \leq \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

$$\sin^3 \theta = \csc \theta$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} \cdot \sin\theta$$

$$\sec\theta \cdot \frac{1}{\csc\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sec\theta}{\csc\theta} = \tan\theta$$

Example 4: Solve over the interval $0 \leq \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

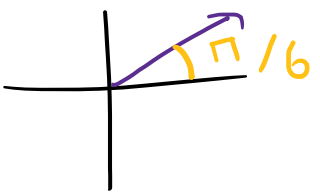
$$2\cot\theta = \sqrt{3}\csc\theta$$

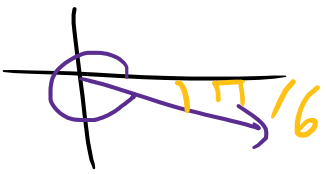
$$2\frac{\cos\theta}{\sin\theta} = \sqrt{3}\cdot\frac{1}{\sin\theta}$$

$$\sin\theta\cdot\left(\frac{2\cos\theta}{\sin\theta}\right) = \left(\frac{\sqrt{3}}{\sin\theta}\right)\cdot\sin\theta$$

$$2\cos\theta = \sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$





$$\theta = \frac{\pi}{6} \qquad \theta = \frac{11\pi}{6}$$

Example 5: Solve over the interval $0 \leq \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

$$2\csc^2\theta = 3\cot\theta\csc\theta + 2$$

$$2\left(\frac{1}{\sin\theta}\right)^2 = 3\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{1}{\sin\theta}\right) + 2$$

$$\sin^2\theta \cdot \left(\frac{2}{\sin^2\theta}\right) = \left(\frac{3\cos\theta}{\sin^2\theta} + 2\right) \cdot \sin^2\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$2 = 3\cos\theta + 2\sin^2\theta$$

$$2 = 3\cos\theta + 2(1 - \cos^2\theta)$$

$A = \cos\theta$

$$2 = 3A + 2(1 - A^2)$$

$$2 = 3\cos\theta + 2 - 2\cos^2\theta$$

$$2\cos^2\theta - 3\cos\theta = 0$$

$$(\cos\theta)(2\cos\theta - 3) = 0$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ $\quad$ $\theta = \frac{3\pi}{2}$

$2\cos\theta - 3 = 0$

$\cos\theta = \frac{3}{2}$

impossible

$A = 0$ $\quad$ $A = \frac{3}{2}$

Example 6: Solve over the interval $0 \le \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

$$\tan\theta = \frac{\sec\theta}{\cot\theta}$$

$\theta = 0 \qquad \theta = \pi$

$\cot\theta \cdot \tan\theta = \cot\theta \cdot \dfrac{\sec\theta}{\cot\theta}$

$1 = \sec\theta$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta}$

$\cos^2\theta \left(\dfrac{\sin\theta}{\cos\theta}\right) = \left(\dfrac{\sin\theta}{\cos^2\theta}\right) \cdot \cos^2\theta$

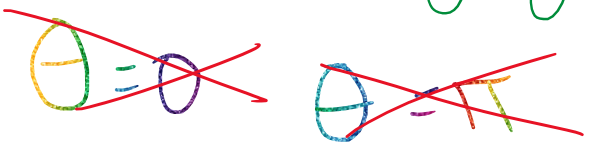
$\cos\theta\sin\theta = \sin\theta$

$\cos\theta\sin\theta - \sin\theta = 0$

$(\sin\theta)(\cos\theta - 1) = 0$

$\sin\theta = 0$ $\qquad$ $\cos\theta = 1$

$\theta = 0 \qquad \theta = \pi$ $\qquad$ $\theta = 0$



No Solution

Other than the Pythagorean identities, reciprocal identities, and quotient identities, here are some tricks we can do to rewrite trigonometric expressions:

Common Denominator

$$\frac{1}{\cos\theta} - \cos\theta$$

Multiply by Conjugate

$$\frac{\sin\theta}{1+\cos\theta}$$

Difference of Squares

$$\frac{1-\cos^2\theta}{1+\cos\theta}$$

Example 8: Three of the following expressions are equivalent. One is not. Find the one that is not equivalent to the other three.

$\sin\theta \tan\theta$

$\dfrac{\sin^2\theta - 1}{\sin\theta}$

$\sec\theta - \cos\theta$

$\dfrac{1 - \cos^2\theta}{\cos\theta}$

Example 9: Three of the following expressions are equivalent. One is not. Find the one that is not equivalent to the other three.

$$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$$

$$\frac{\tan\theta - 1}{\tan\theta + 1}$$

$$\frac{1 - 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$$

$$\frac{2\csc\theta\sec\theta - 1}{\tan^2\theta - \cot^2\theta}$$

Example 10: Solve over the interval $0 \leq \theta < 2\pi$. Be careful of values that are outside the domain of these functions.

$$\sec^3\theta - \tan\theta\sec\theta = 3\sec\theta + \cos\theta$$